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Flux pumping, fluctuations and climbing fields

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Abstract
This paper describes the behaviour of bulk superconductors when subjected to a varying magnetic field. A magnetic model is described together with experimental results which explain and describe the behaviour of superconducting bulks when subjected to varying magnetic fields. We demonstrate how the behaviour is dependent on the magnitude and period of the perturbations in the fields. The model which we use has been implemented using the Comsol^{TM} pde solver. It is a fully integrated model which uses a variable heat source to regulate the magnetic circuit and thereby to achieve flux pumping. Comsol^{TM} is used for post solution visualization and the model is presented alongside experimental results which support and confirm the conclusions from the model.

(Some figures may appear in colour only in the online journal)

1. Introduction

Bulk superconductors have high critical currents and have been magnetized to very high magnetic flux densities [1–3]. So high in fact that Tomita *et al* [1] had to employ steel reinforcement to prevent the superconductor disintegrating due to the Lorentz forces.

We have been investigating methods to magnetize these superconductors *in situ* and in practical machines in order to fully utilize the very high magnetic fields which they are capable of providing [4, 5]. The ideal magnetization method is one in which the magnetization fixture does not dominate the machine design.

Flux pumping which involves varying the reluctance of a magnetic circuit in which the superconductor is placed shows great promise [6, 7]. This paper further explores the mechanisms which enable flux pumping to work and presents both measured data showing how flux pumping progresses and results obtained from a model constructed in Comsol, this model is an extension of the one which was reported on in [8].

Using Bulk Superconductors magnetized by flux pumping opens up a wealth of possible applications including but not limited to: accelerator magnets, MRI magnets, ultra-compact motors for cars, trains and ships, generators for wind and wave power and magnets for magnetic separation and purification of waste.

Varying the reluctance of the magnetic circuit is achieved using ferro- or ferri-magnetic materials which are heated and cooled to above and below their Curie points. A particularly useful material is gadolinium which has a Curie point of 23C and therefore lends itself to easy experimentation as has been reported on in [9]. There are a range of materials which could be used including analogues of Prussian Blue [10] and Dysprosium which has a Curie point just above 77 K and the various manganites and ferrites which have Curie points over a range of temperatures at or around room temperature.

2. Theory

2.1. Equation system

In order to predict the behaviour of the system we have created two models. The first uses COMSOL alone and is based on the so called H formulation [11]. The electrical part of this model is based on the Maxwell–Faraday law (1) together with an E–J current law (2) which describes the behaviour of the superconductor.

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}} \qquad (1)$$

$$E_z = E_0 \left(J/J_c\right)^n \tag{2}$$

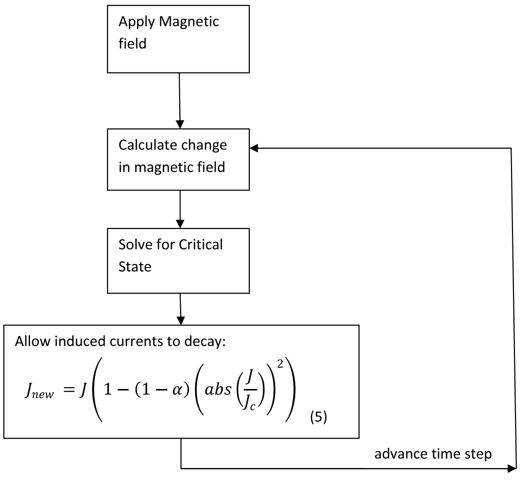
$$J = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{3}$$

The process we are using relies on the change in magnetic field which occurs as a result of a magnetic material's permeability change. In addition we assume that the electric field is perpendicular to the magnetic field and parallel to the flow of current. The axis system in the model assumes that x and y are in the plane of the representation and z is into the paper. Hence equation (1) can be rendered as:

$$\frac{\partial \boldsymbol{E}}{\partial z} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} - \boldsymbol{H}\frac{\partial \mu}{\partial t}. \tag{4}$$

This form of the equation is programmed directly into COMSOL with $\boldsymbol{H}$ rendered as its two components $\boldsymbol{H}_x$ and $\boldsymbol{H}_y$.

The second scheme uses an extension to an algorithm first proposed by Coombs *et al* [12] in which the vector potential A is used. In this algorithm currents are added iteratively within the superconductor at the position of maximum of the modulus of the vector potential in a sense so as to reduce the magnitude of the modulus of the vector potential. This algorithm has been extended to take into account the temporal decay of the currents within the superconductor by allowing the magnitude of the current to decay exponentially. This is achieved using the decay function defined in equation (5) and applied at each timestep according to the flow diagram shown in figure 1.



Currents are initially induced at $J = J_c$ and α = initial decay rate

Figure 1. Flow diagram for solution method.

2.2. Mechanism

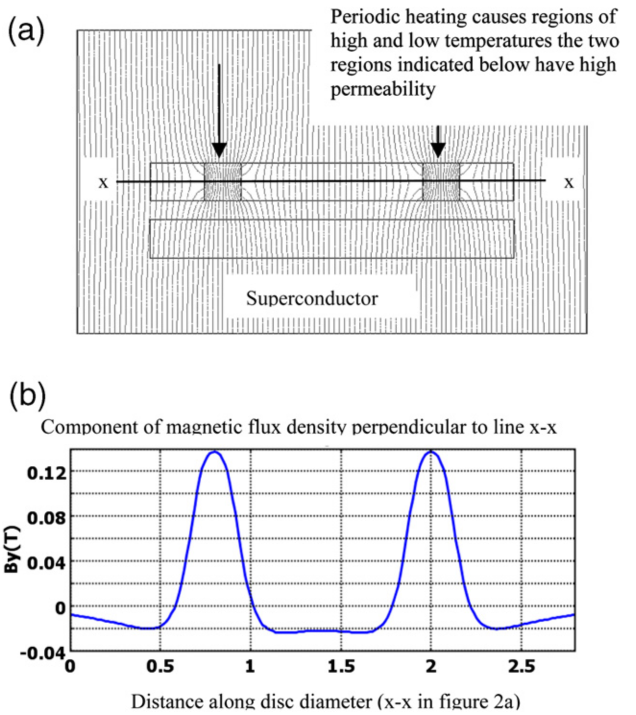


Figure 2. Field profile for a ferromagnetic disc heated at its perimeter and positioned above a superconductor. Note the superconductor has been field cooled.

Flux pumping uses the change in permeability of a magnetic material with temperature to produce a magnetic wave. Local disturbances in the permeability of the magnetic material produce local changes in the magnetic flux density. If you take a circular puck of some ferromagnetic material and heat it on its rim then the heat will diffuse into the centre of the puck as a 'thermal wave'. Since the permeability of the puck is changing with temperature there will also be an associated 'magnetic wave'. We can predict what this waveform will look like by considering Gauss's law:

$$\nabla \boldsymbol{B} = 0. \tag{6}$$

According to Gauss's law since the divergence of B is zero the magnetic wave will have a peak in its centre and a trough in front and behind of it. This can be easily seen from figure 2.

In figure 2 there is a disc above a superconductor. Regions in the left- and right-hand halves of the disc have been given high permeability to signify the change in permeability due to temperature and the whole assembly has been placed in a uniform magnetic field. To the left and right of the regions of high permeability the flux lines are widely spaced and in the material itself they are bunched together. This leads to a flux profile along the top of the superconductor as is shown in figure 2(b). In the figure we have plotted the magnitude of delta B_y. Delta B_y is defined as the change in magnetic flux density due to the change in permeability of the magnetic circuit. Thus there may be a background field of, for example, 1 T and increasing the permeability locally raises it to 1.1 T in this case then delta B_y would be 0.1 T. This is done because the superconductor responds to changes in flux density.

Flux pumping works because, according to the Maxwell–Faraday law coupled with an E–J current law, currents are induced in the superconductor which are a function of the rate of change of the local magnetic flux. It was shown in [8] that a travelling magnetic peak will induce an electric field. As can be seen in figure 8(b) the positive peaks associated with the regions of high permeability are accompanied by much smaller but broader negative peaks.

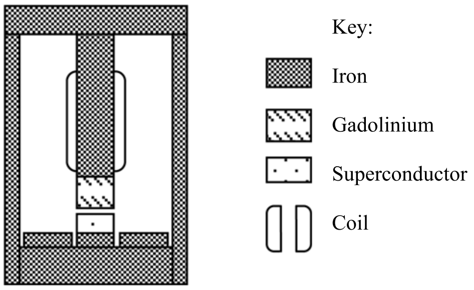


Figure 3. Overall arrangement. Magnetic field is provided by coils via a magnetic circuit composed of iron and gadolinium to a superconductor.

The electric field induced by the positive peaks is therefore greater than that induced by the negative peaks. In the pure Bean model the value of J_c depends only on the presence of an electric field not its magnitude. If an *E–J* current law is used, however, the magnitude of the currents are dependent on the magnitude of the electric field. Once induced these currents decay. The rate of decay is a function of the effective resistivity which can be derived from the *E–J* current law. Therefore the positive currents induced by the positive peaks and the negative currents induced by the negative peaks are different and have different decay rates when an *E–J* current law is used. When a pure Bean model is used the positive and negative currents are identical and all changes add up to zero over a cycle, when an *E–J* current law is used this is no longer the case. In consequence the magnetization of the superconductor can change with each cycle and the superconductor can be magnetized as the modelling and measurements show.

There are a range of magnetic materials available to modulate the magnetic field. In the experiments and the model gadolinium has been used which has a relative permeability μ_r which varies with temperature T according to equation (7).

$$\mu_r = \mu_{r_{\min}} + \frac{\alpha}{(1+e^{-\frac{(T-T_0)}{\beta}})^{\gamma}} \tag{7}$$

where:

$$\mu_{r_{\min}} = 1.306; \qquad \alpha = 12.9685;$$
$$T_0 = 290.04\ \mathrm{K}; \qquad \beta = -4.3854; \qquad \gamma = 0.4811.$$

3. Geometry

The results presented here show the progressive magnetization of a superconductor placed in a magnetic circuit which is regulated by changing the permeability of gadolinium.

The rest of the magnetic circuit is iron with a relative permeability of 1000. The applied magnetic field is provided by a coil wrapped around the central leg of the magnetic circuit. In order to ensure that the output of the model shows the magnetization of the superconductor and only the magnetization of the superconductor the resistivity of all parts of the model apart from the superconductor has been set to $10^7\ \Omega$ m. This eliminates the effects of eddy currents in the iron. The whole arrangement is shown in figure 3.

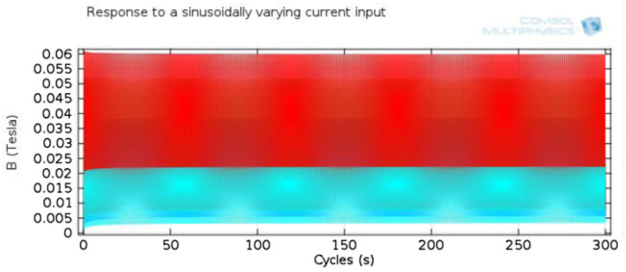


Figure 4. Response to a field being applied and removed 300 times.

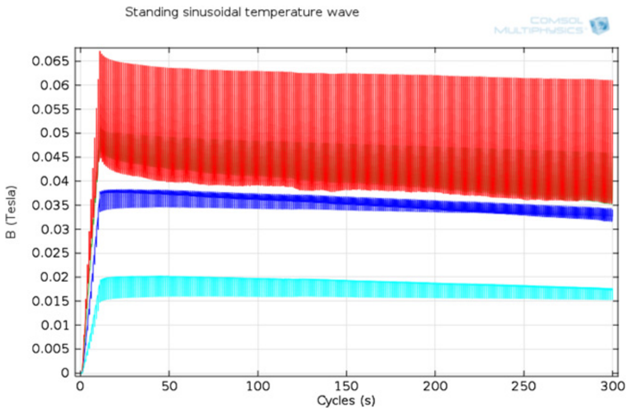


Figure 5. Response to the permeability of the gadolinium region being raised and lowered 300 times.

4. Modelling results

Two models have been constructed both of which use the Comsol solver. The first model which uses Comsol 4.2a is based on the *H* formulation [13]. The second model uses Comsol 3.5a and the vector potential and has been reported on in [8]. The two models produce similar results but in this paper are used for two different purposes. The Comsol 4.2a model has been used to demonstrate that simply changing the magnitude of an applied field will not produce flux pumping but that a radially swept field will produce pumping. The second model has been used to investigate how the pumping effect changes with the magnitude of the modulation of the applied field.

The 4.2a model was tested using three different scenarios in each of which the superconductor sees a varying field.

Scenario 1. Current in the coil is oscillated up and down. This scenario is used to test the model. It corresponds to the scenario which is encountered during pulse magnetization.

Scenario 2. The temperature of the gadolinium is varied uniformly and periodically between 300 K and 340 K. Nominally this is the same as scenario 2 since the nett effect is to periodically raise and lower the applied field to the superconductor. However in scenario 1 the amplitude of the oscillation is fixed whereas the amplitude of the oscillation in scenario 2 is dependent on the total flux in the circuit and this will vary with the magnetization of the superconductor.

Scenario 3: The temperature of the gadolinium is varied using a travelling wavefunction. This simulates what happens in the

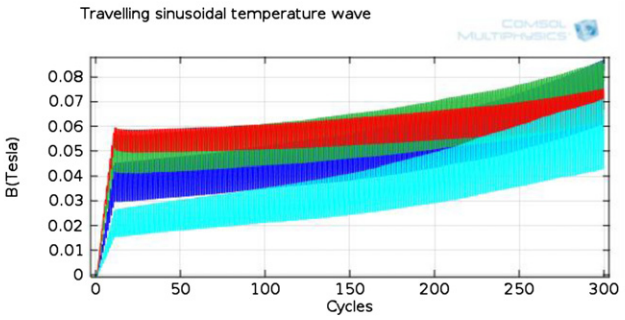


Figure 6. Response to a radially travelling temperature wave modulating the permeability of the gadolinium region.

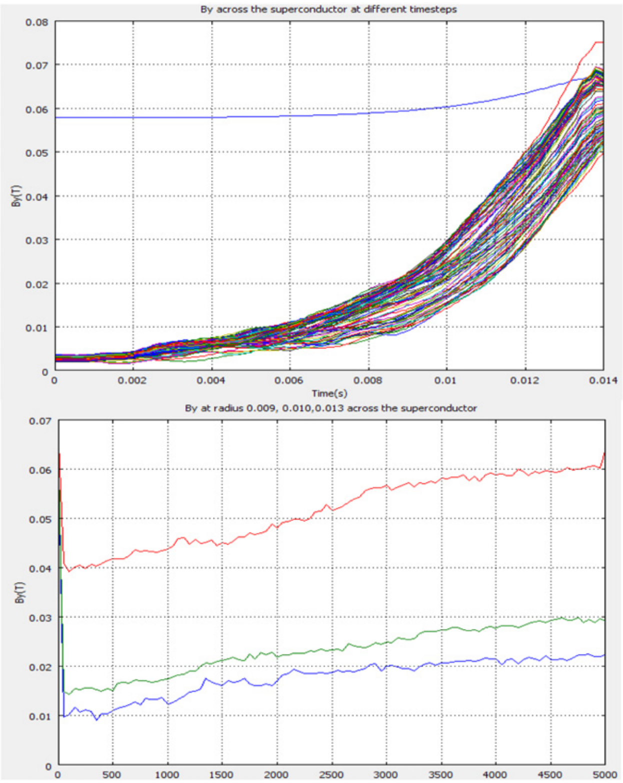


Figure 7. Response to modulation of a small (approximately 0.06 T) background field.

experiment when the perimeter of the gadolinium is heated and the heat diffuses into the centre of the gadolinium. This is the basic mechanism of flux pumping.

Figures 4–6 show the results obtained from the model for the three different scenarios, plotted at four discrete points across the superconductor (to match the positions of the Hall probes in the experimental results given below).

Several things are evident. The first is that simply oscillating the magnitude of the magnetic field which the superconductor is subjected to does not lead to a progressive magnetization. This is true whether the field is modulated directly by varying the current in the coil or indirectly by changing the reluctance of the magnetic circuit.

However, changing the reluctance of the magnetic circuit using a radially travelling temperature wave such as would be induced when the perimeter of a gadolinium puck was repeatedly heated and then cooled does produce pumping.

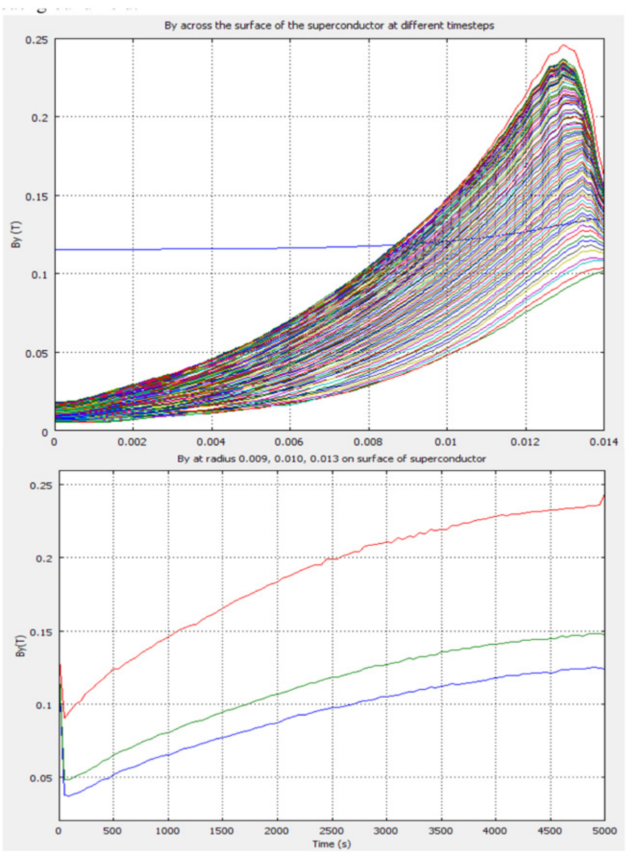


Figure 8. Response to modulation of a medium (approximately 0.1 T) background field.

In addition it can be seen that since the magnetic circuit is being modulated that the amplitude of the oscillations increases with each cycle as the total magnetic field in the circuit increases. This effect helps the flux to penetrate ever deeper into the superconductor and to eventually fully magnetize the superconductor.

While the 4.2a model was used to investigate the effect of different mechanisms of modulating the magnetic field, the 3.5a model was tested using three different magnitudes of modulation applied field. The results which are presented in figures 7–9 show a clear progression from a small field which will produce a small change in current in the superconductor but in which the current profile does not penetrate radially. To a large field in which the current clearly penetrates radially and in which the behaviour is categorized by the position of the peak magnetic field moving radially inwards as the superconductor becomes more and more magnetized. This behaviour is observed in the experimental results which are summarized in section 5.

5. Experimental results

The current experimental arrangement is shown part assembled in figure 10. A magnetic circuit is formed by the top and bottom plates and the side and centre rods.

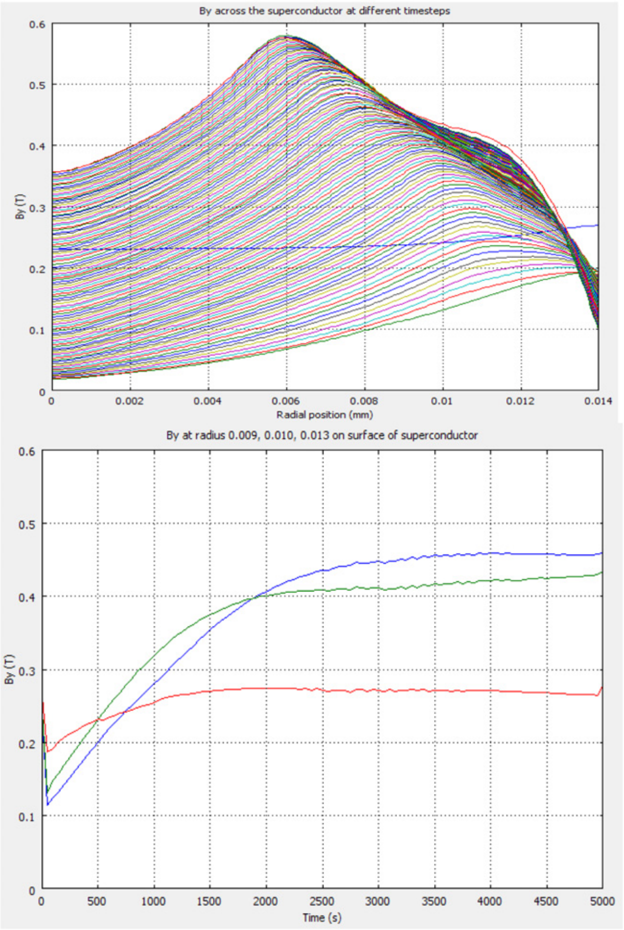


Figure 9. Response to modulation of a large (approximately 0.2 T) background field.

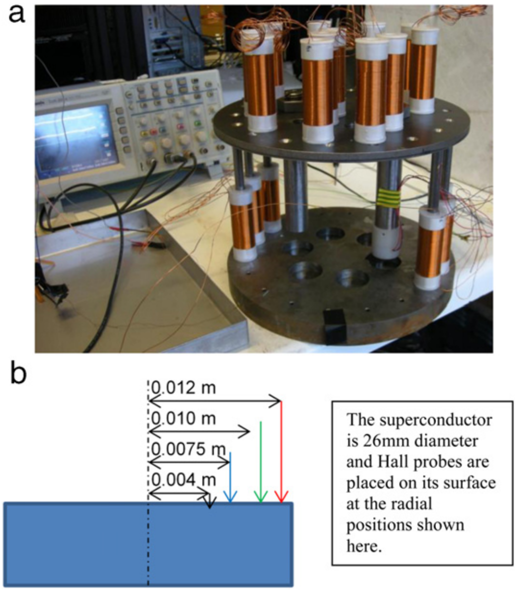


Figure 10. (a) Experimental arrangement. (b) Positioning of Hall probes.

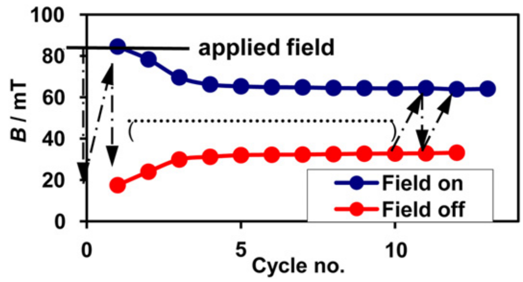


Figure 11. Response to a field being applied and then removed.

Magnetic field is applied using coils and the gadolinium is held in place above the superconductor in the white nylon sheath which can be seen on the right-hand side of the diagram. Superconductors are placed in the indentations seen in the bottom plate. The rig is fully instrumented but of particular note is the position of the Hall probes with respect to the superconductor which is shown in figure 10(b) in the experimental results that follow the graphs use the same colour coding as in figure 10(b). The rig can hold up to six superconductors at a time the results presented here are for a single superconductor of 26 mm in diameter which was provided by Krabbes.

Several experiments were performed using this rig. The first experiment was to investigate the effect of cycling the applied field and the results are shown in figure 11. As expected this replicated the results which were obtained from the model and which were presented in figure 4.

Figure 12 shows the response to a medium sized background field. Pumping can clearly be seen as over time the flux density increases. However crucially the relative magnitudes of the Hall probe readings remains the same. This corresponds to the result observed from the modelling in figure 8.

Figures 13 and 14 show what happens when the background field and hence the modulation is increased. In this case the field has only been increased by 10% over that used in figure 12 but nevertheless the behaviour has changed and now corresponds to that observed in figure 9 where there is clear penetration of flux radially into the centre of the superconductor as evidenced by the change in the relative magnitude of the three Hall probes. This penetration can happen over just a few cycles as is seen in figures 13(a) and (b).

In figures 14(a) and (b) the overall background field has been reduced but the modulation has been increased by raising the temperature of the gadolinium. Figure 14(a) demonstrates that flux pumping can be used to both pump down the field (left-hand side of the figure (to 97 000 s) and to pump up the field (97 000–10 700 s)). As the overall field reduces then the magnitude of the oscillation reduces and vice versa.

Figure 14(b) shows the progression of the pumping plotted against the measured temperature of the gadolinium at three different radial points across the superconductor. Also shown on the figure is the applied magnetic field at the three points. the same rapid change as was observed in figure 13 can be seen in the Hall probes at radius 7.5 mm and 10 mm. In addition the field can be seen to carry on climbing.

Finally figure 15 shows the effect of very slow cooling of the gadolinium. In this figure there is an initial phase of

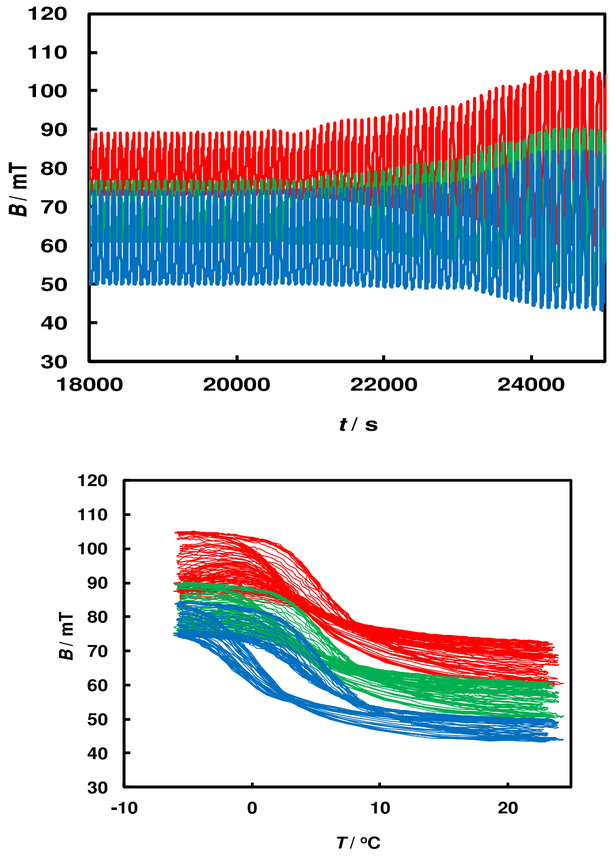


Figure 12. Response to modulation of a medium sized background field.

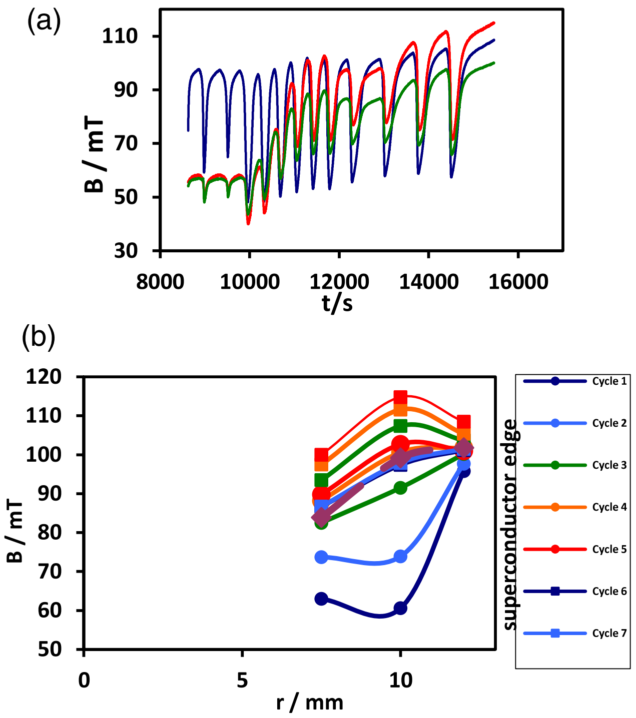


Figure 13. (a) Measured field versus time. (b) Field distribution versus cycle number.

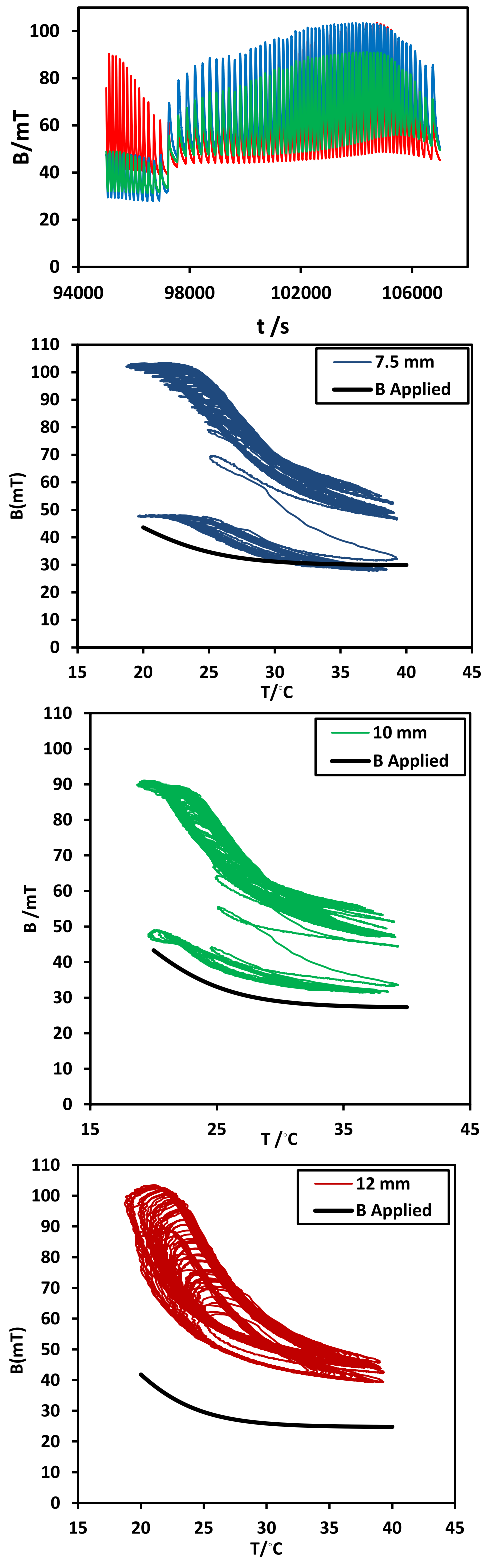


Figure 14. (a) Measured field versus time. (b) Measured field versus gadolinium temperature over time span covered by (a).

pumping up until 3000 s. The gadolinium is then gradually cooled. Since the gadolinium is cooling the overall reluctance of the magnetic circuit is reducing and the 'applied field' is increasing. This can be seen in the measurements obtained

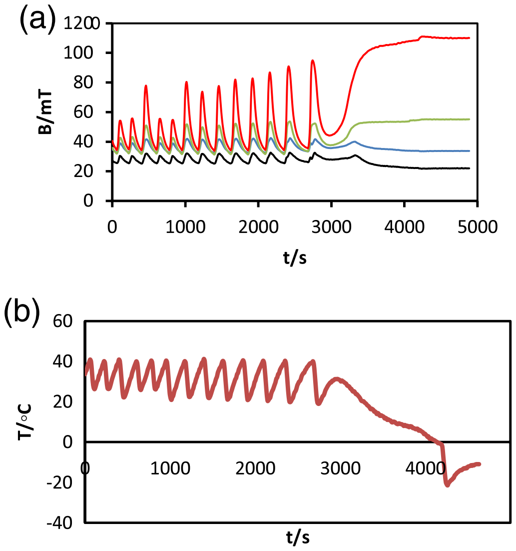


Figure 15. (a) Magnetic field versus time. (b) Gadolinium temperature versus time.

from the two outermost Hall probes. Remarkably though, the two innermost probes show a reducing field, indicating that, in these positions at least, the rate of change of the magnetization of the superconductor was more than the rate of change of the magnetic field.

6. Conclusion

This paper presents two modelling schemes together with experimental results which clearly show that a travelling magnetic wave can be used to magnetize a superconductor. At no stage in the measurements was the applied field sufficient to drive the superconductor normal and the temperature of the superconductor was maintained at approximately 77 K throughout the experiments. We have demonstrated that flux pumping can be used to either magnetize or demagnetize the superconductor and that it is possible to produce a measured magnetic field which is greater than the applied magnetic field.

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